Outage Probability Analysis in Finite Wireless Networks Operating in LOS Environment

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***Abstract*—In this work, we analyze the outage probability in finite wireless networks operating in line-of-sight (LOS) environment with a given statistical distribution of transmitter-receiver distance. To generalize our analysis, we use a recently proposed beta distribution as a substitute to real distance distributions in finite random networks [1]. Effects of generalized LOS Rician fading distribution and path-loss effects are taken into consideration. Moreover, we also analyze effects of possibly different path-loss exponents for LOS and non-LOS fading components [2]-[3].**

***Index Terms*—Fading, finite wireless networks, generalized Rician distribution, line-of-sight propagation, outage probability, random distance.**

I. Introduction

The outage probability (OP) is a very important characteristic of a wireless network affecting many its functionalities. The OP characterizes statistically the node connectivity and thus it plays an important role in design of wireless networks, for instance, in cell planning. The OP statistical distribution gives a total statistical characterization of the signal-to-interference ratio (SIR) affecting many important performance metrics of wireless networks. Meanwhile, OP analyzing under practical wireless conditions including fading, path-loss, and interference effects, is a rather challenging problem.

The OP in wireless networks was analyzed, for example, in [4], [1].

In this work, we consider the problem of OP statistical characterization in finite wireless networks where path-loss and line-of-sight (LOS) propagation effects are taken into account. Nowadays, a large interest to analyzing of LOS environment is caused by potential exploiting millimeter wave (mmWave) communications in 5G networks [5]. It is well known that the presence of dominant LOS is a significant feature of mmWave propagation conditions due to their quasi-optical propagation characteristics [2]. We use generalized Rician distributions [6] (also known as κ-μ distributions [7]) for modeling of fading effects. This fading model exhibits very good fits to measuring data in LOS environment [7].

Including path-loss effects into consideration requires a knowledge on statistical models of transmitter (Tx)-receiver (Rx) distance. It depends on many factors such as a shape of operating area and node distribution inside the area. Statistical models of distances between two arbitrary nodes were reported for some operating areas, see, for example, [8]- [9]. Derived expressions for the probability density and cumulative distribution functions (PDFs and CDFs) are often represented by rather complicated expressions, which makes their further application challenging. In this work, we employ a recently proposed substitute of the beta distribution for real statistical models of distances in finite random networks [1]. This substitute may result in tractable performance analysis, and additionally it generalizes an analysis since a uniform parametric solution for many operational scenarios can be obtained.

Recent works [2]- [3] based on real measurements revealed that LOS and non-LOS (NLOS) signal components may be subject to different path-loss attenuations. In this work, we analyze this effect too.

The paper is organized as follows. In Section II, we present statistical models of wireless propagation and distances between two arbitrary network nodes. The OP analysis is given in Section III, Section IV presents numerical estimates, and Section V concludes the work.

II. System Model

We assume that the maximal Tx-Rx distance in the considered finite network is $R_{\max}$. Below, we present path-loss, fading, and statistical distance models.

A. Path-loss Model

We apply a conventional distance-dependent path-loss model [Gold].The path-loss signal power attenuation γ_{PL} is specified by the Tx-Rx distance R as

$$\gamma_{\mathrm{PL}} = K_0 \left(\frac{R}{R_{\max}} \right)^{-\lambda} \tag{1}$$

where λ is the path-loss exponent, and K_0 denotes the value of γ_{PL} at $\frac{R}{R_{\max}} = 1$. In the typical environment, $\lambda \in [2, 6]$ [10].

B. Generalized Rician ($\kappa - \mu$) Fading Distributions

The generalized Rician fading distributions (also known as the κ-μ distributions) model well random signal fluctuations in LOS conditions [6], [7]. The physical model of the generalized Rician distribution assumes that the fading signal is a composition of multipath cluster with non-zero mean Gaussian components within each cluster, that is the signal power can be represented via the in-phase (I) and quadrature (Q) components of fading signal as

$$W = \sum_{i=1}^{n} (X_i + p_i)^2 + (Y_i + q_i)^2 \quad (2)$$

where n is a number of multipath clusters, X_i, Y_i are the respective I and Q zero-mean Gaussian components with the variance $\sum_{i=1}^{n} E\{X_i^2\} + E\{Y_i^2\} = 2\sigma^2$ with E denoting the expectation. The variables p_i, q_i represent the dominant components of fading signal with the total power $\sum_{i=1}^{n} p_i^2 + q_i^2 = d^2$.

The PDF $f_{\gamma_{\rm fad}}(x)$ of the power variable $\gamma_{\rm fad}$ is given in [7] as

$$f_{\gamma_{\rm fad}}(x) = \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}} x^{\frac{\mu-1}{2}}}{\kappa^{\frac{\mu-1}{2}} \exp(\mu\kappa) \Omega^{\frac{\mu+1}{2}}} \exp\left(-\frac{\mu(1+\kappa)x}{\Omega}\right) \times I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(1+\kappa)x}{\Omega}}\right) \quad (3)$$

where $\kappa > 0$ is the ratio of the total power of the dominant components to that of the scattered waves, $I_\nu(x)$ is the Bessel I function of the order ν [11]. In (3, $\Omega = E\{\gamma\} = \gamma_{\rm T} 2\sigma^2(1+\kappa)$ where $2\sigma^2$ is the total power of scattered waves, and $\gamma_{\rm T}$ is the transmit signal-to-noise ratio (SNR). The parameter μ is a real-valued extension of n in (2). It characterizes the number of multipath clusters. Due to physical reasons [7], μ can take on non-integer values.

The CDF of $\gamma_{\rm fad}$ can be specified as [7]

$$F_{\gamma_{\rm fad}}(z) = 1 - Q_\mu\left[\sqrt{2\kappa\mu}, \sqrt{\frac{2(1+\kappa)\mu z}{\Omega}}\right] \quad (4)$$

where $Q_\mu(a, b)$ is the generalized Marcum function [12].

Three parameters of (3)-(4) provide a better fit to experimental data than do commonly used fading models. Due to physical reasons, a fitting procedure may result in non-integer values of μ [7].

C. Approximations of Real Distance Distributions by Beta Distributions

Based on the fact that real distance distributions in finite random networks are characterized by finite supports, a beta distribution was proposed in [1] to substitute the real statistical model. According to [1], the PDF of normalized Tx-Rx distance $X = R/R_{\max}$ can approximately be represented as

$$f_X(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)} \quad (5)$$

where α and β are the distribution parameters, $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ is the beta function, and $\Gamma(.)$ is the gamma function. α and β are defined using the moment-matching approach [] by equating two first moments of real distance distribution to those of beta distribution, see also [1] for details. Examples given in [1]show a good approximation accuracy for different shapes of operational areas and nodes distributions inside the areas.

III. OP Analysis

The OP $P_{\rm out}(\gamma_0)$ is defined as the probability that the SIR is below a predetermined level γ_0, that is it is totally defined by a SIR statistical model. We consider both noise-limited and interference-limited scenarios. First, we analyze conventional path-loss models where the path-loss attenuation is the same for Los and NLos signal components

A. Equal Path-Loss Exponents for LOS and NLOS Signal Components

1) Noise-limited environment: Under this scenario, the total signal power attenuation caused by fading and path-loss effects can be factorized into the product of two independent components, and the OP can be expressed as

$$P_{\rm out}(\gamma_0) = Pr\{\gamma_{\rm fad}\gamma_{\rm PL} \le \frac{\gamma_0}{\gamma_{\rm T}}\} = 1 - E_X\left\{Q_\mu\left[\sqrt{2\kappa\mu}, \sqrt{\frac{2(1+\kappa)\mu X^\lambda \gamma_0}{\gamma_{\rm T} K_0 \Omega}}\right]\right\} \quad (6)$$

where (1) and (4) were used to specify $\gamma_{\rm PL}$ and CDF ($\gamma_{\rm fad}$), respectively.

For some fading conditions, numerical integration can be avoided with the help of following proposition.

Proposition 1: If the path-loss exponent $\lambda = m/k$ where m and k are integers, then $P_{\rm out}(\gamma_0)$ can approximately be evaluated in a closed form as

$$P_{\rm out} \approx \frac{\exp(-\kappa\mu)\,\Gamma(\alpha+\beta)}{\Gamma(\alpha)} \sum_{l=0}^{N_{\max}} \frac{(\kappa\mu)^l}{l!\Gamma(l+\mu)} \times \frac{k^{l+\mu-1/2} l^{-\beta}}{(2\pi)^{(k-1)/2}\Gamma(l+\mu)} G_{k+m,2k+m}^{k,k+m} \left[\left(\frac{\theta}{k}\right)^k \middle| \begin{array}{c} \Delta(m, 1-\alpha) \\ \Delta(k, l+\mu), \Delta(m, 1-\alpha-\beta) \end{array}\right]. \quad (7)$$

where $G[.]$ is the Meijer G function [13, vol. 3], $\theta = (1+\kappa)\mu\gamma_0/(K_0\Omega\gamma_{\rm T})$.

The accuracy of (7)

$$\epsilon < P(N_{\max}\mu\kappa) \quad (8)$$

where $P(N_{\max}\mu\kappa) = \gamma(N_{\max}, \mu\kappa)/\Gamma(N_{\max})$ is the lower regularized gamma function, and $\gamma(c, x) = \int_0^x t^{c-1}\exp(-t)dt$ is the lower incomplete gamma function.

Proof: We apply a series expansion of $Q_\mu(a, b)$ in the variable a given in [14] as

$$Q_\mu(a, b) = \exp\left(-\frac{a^2}{2}\right) \sum_{l=0}^{\infty} \frac{a^{2l}\Gamma\left(l+\mu, \frac{b^2}{2}\right)}{2^l l! \Gamma(l+\mu)} \quad (9)$$

where $\Gamma(c, x) = \int_x^\infty t^{c-1}\exp(-t)dt$ is the upper incomplete gamma function. Then using (5) and applying Tonelli's theorem [15], we find that

$$P_{\rm out} = 1 - \frac{\exp(-\kappa\mu)}{B(\alpha, \beta)} \sum_{l=0}^{\infty} \frac{(\kappa\mu)^l}{l!\Gamma(l+\mu)} \int_0^1 x^{\alpha-1}(1-x)^{\beta-1} \times \Gamma\left[l+\mu, (1+\kappa)\mu x^\lambda \gamma_0/(K_0\Omega\gamma_{\rm T})\right] dx. \quad (10)$$

The integral in (9) can be evaluated in a closed form if $\lambda = m/k$ where m and k are integers. Expressing the incomplete gamma function in (8) via the Meijer G function as $\Gamma(a,x) = \Gamma(a) - G^{1,1}_{1,2}\left[x \,\middle|\, \begin{matrix} 1 \\ a, 0 \end{matrix}\right]$ [13, vol. 3, (8.4.16.1)] and using an integration formula [13, vol. 3, (2.24.2.2)], we obtain a formula, which is identical to (7) with the difference that the series is infinite. Since the evaluation of infinite series requires its truncation, the residual must be assessed. In our case, this can be done by assessing the residual of (7) and taking into account the integrand and integration limits in (9). Let $(N_{\max}+1)$ terms be kept in (8).

A factor in (9) $\frac{\Gamma(\mu+l,.)}{\Gamma(\mu+l)} < 1$, and thus the convergence of (9) can be defined by the other factor representing the upper regularized gamma function $Q(N_{\max}, \mu\kappa) = \Gamma(N_{\max}, \mu\kappa)/\Gamma(N_{\max})$. Thus, the series residual is defined as $1 - Q(N_{\max}, \mu\kappa) = P(N_{\max}, \mu\kappa)$.

The remainder of infinite series expressing P_{out}, $R_{N_{\max}}$, is a result of averaging of $r_{N_{\max}}$ over the PDF (5) and therefore $R_{N_{\max}} = r_{N_{\max}}$, and (8) follows. ■

The Meijer G function is implemented in many standard software such as Mathematica and Maple.

2) Interference-limited scenarios with Poisson fields of interferers: For this scenario, we restrict our consideration by integer values of fading parameter μ. We analyze interference-limited scenarios with interference I coming from Poisson fields of interferers characterized by the moment generating function (MGF) of interference $M_I(-s) = E\{\exp(-sI)\}$ derived in [16]- [18] as

$$\mathcal{M}_I(-s) = \exp\left(-Ks^{\frac{2}{\eta}}\right) \tag{11}$$

where K is defined by network and fading parameters with explicit expressions given in [16]- [18] for different operational scenarios.

In this case, the OP can be evaluated as

$$\begin{aligned} P_{\text{out}}(\gamma_0) &= 1 - E_{I,r}\left\{Q\left(\sqrt{2\kappa\mu}, \sqrt{\frac{2\gamma_0 I r^{\lambda}(1+\kappa)\mu}{K_0\gamma_{\text{T}}\Omega}}\right)\right\} \\ &\approx 1 - \exp(-\mu\kappa)\left[\sum_{l=0}^{N_{\max}} \frac{(\mu\kappa)^l}{l!} E_{I,r}\left\{\exp(-\vartheta r^{\lambda} I)\right.\right. \\ &\quad \times \left.\left.\sum_{j=0}^{l} \frac{(-\vartheta r^{\lambda} I)^j}{j!}\right\}\right] = 1 - \frac{\exp(-\mu\kappa)}{B(\alpha\beta)}\left[\sum_{l=0}^{N_{\max}}\sum_{j=0}^{l}\right. \\ &\quad \left.(-1)^j \int_0^1 r^{\alpha-1}(1-r)^{\beta-1}\frac{d^j}{ds^j}M_I(-s)|_{s=\vartheta r^{\lambda}}dr\right] \end{aligned} \tag{12}$$

where $\vartheta = \frac{2\gamma_0(1+\kappa)\mu}{K_0\gamma_{\text{T}}\Omega}$ and $N_{\max}$ defined with the help of (8) provides the accuracy given by (8). To derive (12), we used Marcum Q function series expansion (9), Tonelli's theorem, the finite series expansion of upper regularized gamma function in (9) (valid for integer values of μ), and differentiation properties of Laplace transform. The derivatives in (12) can easily be implemented via the differentiation operator implemented in Mathematica and Maple.

B. Non-equal Path-Loss Exponents for LOS and NLOS Signal Components

Under this scenario, the factorization of joint attenuation caused by fading and path-loss effects as it was done in (6) is not valid more. Taking into account physical meaning of parameters of generalized Rician distribution, see (2)-(3), one can note that (3)- (4) stay valid if to change the parameters κ and Ω to $\hat{\kappa}$ and $\hat{\Omega}$, respectively, as

$$\hat{\kappa} = \frac{K_{\text{Los}}}{K_{\text{NLos}}}\kappa X^{\lambda_{\text{NLos}} - \lambda_{\text{LOS}}} \tag{13}$$

and

$$\begin{aligned} \hat{\Omega} &= \gamma_{\text{T}} K_{\text{NLos}} X^{-\lambda_{\text{NLos}}} 2\sigma^2(1+\hat{\kappa}) \\ &= \frac{\Omega K_{\text{NLos}} r^{-\lambda_{\text{NLos}}}(1+\hat{\kappa})}{1+\kappa} \end{aligned} \tag{14}$$

where λ_{NLos}, λ_{Los} are the respective path-loss exponents for NLOS and LOS signal components, and K_{NLos}, K_{Los} are the respective values of γ_{PL} at $R/R_{\max} = 1$ for NLOS and LOS signal components.

In this case, $P_{\text{out}}(\gamma_0)$ can be evaluated numerically as

$$\begin{aligned} &P_{\text{out}}(\gamma_0) = 1 - E_X \\ &\left\{Q_\mu\left[\sqrt{2\mu\hat{\kappa}X^{\lambda_{\text{NLos}} - \lambda_{\text{LOS}}}}, \sqrt{\frac{2\mu X^{\lambda_{\text{NLos}}}\gamma_0(1+\hat{\kappa}}{\gamma_{\text{T}} K_{\text{NLos}}\hat{\Omega}}}\right]\right\} \end{aligned} \tag{15}$$

IV. Numerical Results

In this section, we present numerical results for different operational scenarios. From physical meanings of parameters κ and μ it is immediately clear that increasing of κ and μ results in decreasing of P_{out}. But the physical meanings only give qualitative estimates, while the techniques given in this work provide quantitative results.

We analysed scenarios considered in Section III and networks with uniform node distributions (UND) within a circle [8] and random waypoint mobility models (RWMMs) operating in a circle, which are characterized by non-uniform node distributions [9]. In the former case, the parameters of the approximating beta distribution are $\alpha = 2.0362$ and $\beta = 2.46185$, while under the latter scenario, $\alpha = 2.34$ and $\beta = 3.97996$ [1]. For all considered cases, the threshold$\gamma_0 = 1$.

In Fig. 1, numerical results are shown for noise-limited scenarios for a few values of κ and μ and for the path-loss exponent $\lambda = 7/2$.

In Fig. 2, numerical estimates are shown for interference-limited scenarios with Poisson fields of interferers Φ_I considered in subsection III.A. We assume that the interferers are subject to Rayleigh fading. The parameter K is defined similarly as in [18, eq.(17)] with the difference that we consider a thinned version of Φ_I appearing due to the fact that each interferer transmits with the probability p. In Fig. 2, the OP curves are shown versus the interferer transmission probability p for a few values of the Tx -to-interferer power ratio $\text{SIR} = P_{\text{T}}/P_{\text{T}}$.

Finally, in Fig. 3, we show numerical results for scenarios where path-loss exponents are different for LOS and NLOS signal components. We use a set of pat-loss exponents given in [2]: $\lambda_{\text{LOS}} = 2.17$ and $\lambda_{\text{NLOS}} = 3.36$. As expected, a smaller LOS path-loss attenuation results in smaller OP values, which is more evident for larger values of κ. The curves in Fig. 3 show numerical estimates evaluated with the help of (15). For all considered scenarios, we observed a very good agreement of numerical estimates with simulation results. This can be explained by good approximation accuracies of beta distributions for considered scenarios, which is a consequence of small values of Kullback-Leibler divergence evaluated in [1].

V. Conclusion

A high interest to LOS propagation conditions is caused by the fact of potential exploiting of mmWave communications in 5G networks, which are characterized by the presence of dominant LOS due to quasi-optical propagation characteristics. Generalized Rician distributions show good match to measuring data, and they are widely accepted as models for LOS propagation scenarios.

In this work, we consider finite wireless networks with arbitrary node distribution operating over areas of arbitrary shape. We analyze the joint effect of generalized Rician fading and path-loss attenuation caused by random Tx-Rx distances on the outage probability. To generalize our consideration, we apply a recently proposed beta distribution as a substitute to real distance models in finite random networks.

Our analysis includes noise-limited scenarios and interference-limited scenarios with the interference coming from Poisson fields of interferers. Moreover, this work also draws attention on recently reported results showing different path-loss attenuation for LOS and NLOS signal components. If this is the case, parameters of LOS fading distributions must be modified. We show how the parameters κ and μ of generalized Rician distributions must be updated and present OP numerical estimates for this scenario.

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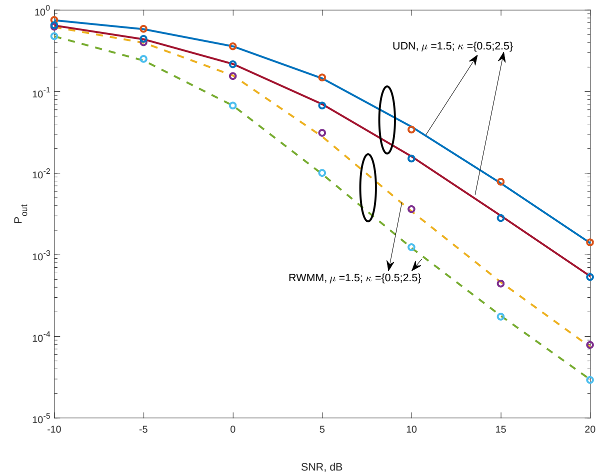

Fig. 1. Outage probability in circular area with UDN and with RWMM. Circles report simulation results

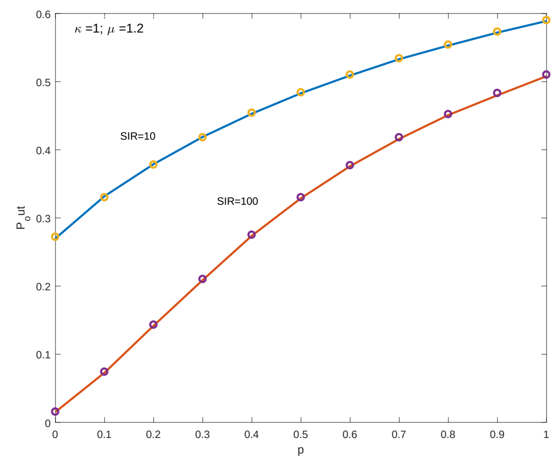

Fig. 2. Outage probability in circular area with Poisson field of interferers versus the interferer transmission probability p. Circles report simulation results.

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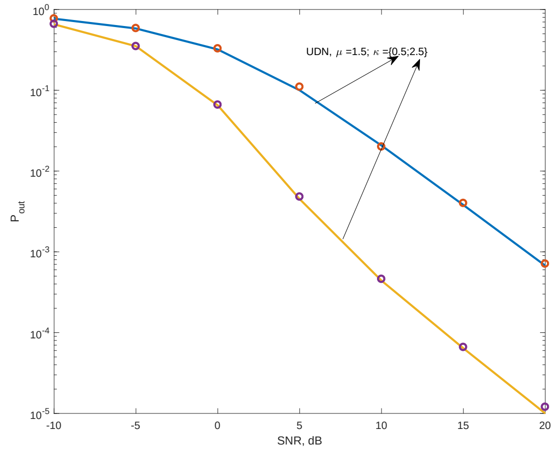

Fig. 3. Outage probability in circular area with UDN for generalized Rician fading model with different path-loss attenuation of LOS and NLOS signal components. Circles report simulation results

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