# Interference Analysis in Wireless Networks Operating over Arbitrary Fading Channels with Heterogeneous Poisson Fields of Transmitters and Interferers

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***Abstract*—In this letter, we analyze statistics of signal-to-interference ratio in wireless networks operating over arbitrary fading channels in interference-limited scenarios with heterogeneous Poisson fields of transmitters and interferers. First, we derive expressions for the moment generating functions of aggregate interference and its negative fractional power. Then we obtain a formula for the outage probability. Depending on an operational scenario, this formula presents either an exact expression or an upper bound. Some obtained results are given in terms of Mittag-Leffler's function. This special function is implemented in standard software, which makes the presented results convenient for analysis and design of wireless networks.**

***Index Terms*—Heterogeneous wireless networks, interference statistics, Mittag-Leffler's function, Poisson point process.**

## I. Introduction

Due to path-loss and fading effects inherent to wireless radio channels, spatial locations of network and interfering nodes and statistics of channel gains are important factors affecting modeling of wireless networks.

A frequent presumption about the spatial distribution of interfering nodes is based on a Poisson point process (PPP) [1]- [9]. PPPs are also appropriate for network (transmitting) node modeling in ad hoc networks. The network nodes generally cooperate [10]- [11]. Recent works [8]- [9] revealed that PPPs can efficiently be applied to modeling of cellular networks too.

Wireless Poisson networks were analyzed in many works such as [1]- [9], [12]- [13], but the considerations were restricted by specific fading distributions of wireless radio channels. Rayleigh fading channels were analyzed most frequently [8]- [9]. Nakagami-$m$ channels were considered in [7]. More sophisticated $\kappa$-$\mu$ shadowed and compound Nakagami-$m$ – log-normal fading models were investigated in [12]- [13].

Transmitting and interfering nodes may coexist within one network and be dependent. Such are cellular networks where a user connects to a base station (BS), and the other BSs are sources of interference [8]- [9], [12]- [13]. Under other operational scenarios transmitting and interfering networks are independent. Such are spectrum sharing networks where one network operates as an underlay system with another [1], [7]. Spectrum sharing is a trend technology for 5G communications [14]- [15]. An important problem in such networks is providing of acceptable interference level. Interference analysis gives the basis for specification of network parameters. Jamming networks (that are generally heterogeneous) represent other examples of autonomous interfering networks [16]. Interference analysis is of interest for these scenarios too.

In this work, we analyze statistics of signal-to-interference ratio (SIR) in wireless networks with heterogeneous Poisson fields of cooperative transmitters (Txs) and independent interferers operating in interference-limited scenarios over arbitrary fading channels. We obtain formulas for the moment generating functions (MGFs) of aggregate interference, its negative fractional power, and for the outage probability (OP). We prove that some SIR statistics can efficiently be assessed in terms of Mittag-Leffler's function. This special function is implemented in standard software, which makes its application convenient.

The remainder of the letter is organized as follows. In Section II, we present the network model and introduce Mittag-Leffler's function. In Section III, we analyze SIR statistics. Numerical results are given in Section IV, and Section V summarizes this work.

## II. Preliminaries

### A. Wireless Network Model

We consider wireless networks operating in the two-dimensional Euclidean space $\mathbb{R}^2$ under interference-limited scenarios. We assume that the network nodes cooperate [10]- [11], and they form a heterogeneous Poisson field $\Theta_{\mathrm{T}} = \{\Theta_{\mathrm{T}_j}\}\Big|_{j=1}^{N_{\mathrm{T}}}$ composed of a set of independent homogeneous PPPs $\Theta_{\mathrm{T}_j}$ characterized by the transmit power $P_{\mathrm{T}_j}$ and density $\lambda_{\mathrm{T}_j}$. An autonomous Poisson field of interferers $\Theta_{\mathrm{I}} = \{\Theta_{\mathrm{I}_j}\}\Big|_{j=1}^{N_{\mathrm{I}}}$ consists of independent homogeneous PPPs $\Theta_{\mathrm{I}_j}$ characterized by the transmit powers $P_{\mathrm{I}_j}$ and densities $\lambda_{\mathrm{I}_j}$. Transmission probabilities of network and interfering nodes can easily be incorporated into the network model resulting in thinned versions of considered PPPs [9]. Similarly to [9], we refer each $\Theta_{\mathrm{T}_j}$ and $\Theta_{\mathrm{I}_j}$ to as a tier of $\Theta_{\mathrm{T}}$ and $\Theta_{\mathrm{I}}$, respectively.

Channel power gains $g_{x_j}$ and $g_{y_j}$ reflect fading effects between $x_j \in \Theta_{\mathrm{T}_j}$ and the receiver (Rx) of interest, and between

Manuscript received June 16, 2017. This work was supported in part by the Finnish Funding Agency for Technology and Innovations under Grant 40156/14 and in part by the European Union under Grant ICT-671639. The associate editor coordinating the review of this manuscript was NN.

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$y_j \in \Theta_{\mathrm{I}_j}$ and the Rx, respectively. We assume that $g_{x_j} \sim \mathcal{F}_{\mathrm{T}_j}$ and $g_{y_j} \sim \mathcal{F}_{\mathrm{I}_j}$ where $\mathcal{F}_{\mathrm{T}_j}$ and $\mathcal{F}_{\mathrm{I}_j}$ are fading distributions with the respective MGFs $\mathcal{M}_{g_{x_j}}(-s) \triangleq E_{g_{x_j}}\{\exp(-sg_{x_j})\}$ and $\mathcal{M}_{g_{y_j}}(-s) \triangleq E_{g_{y_j}}\{\exp(-sg_{y_j})\}$, where $E_g$ means the expectation with respect to $g$. The probability density functions (PDFs) of $\mathcal{F}_{\mathrm{T}_j}$ and $\mathcal{F}_{\mathrm{I}_j}$ are $f_{g_{x_j}}(x)$ and $f_{g_{y_j}}(x)$, respectively, and the corresponding cumulative distribution functions (CDFs) we denote as $F_{g_{x_j}}(x)$ and $F_{g_{y_j}}(x)$.

We apply a standard path-loss model specified as [19]

$$l(x) = L_0||x||^{-\eta} \tag{1}$$

where $||.||$ means the Euclidean distance between the Tx and Rx, $L_0$ represents the path loss at $||x|| = 1$, and $\eta$ is the path-loss exponent. Typically, $2 < \eta \le 6$ [19].

### *B. Mittag-Leffler's function*

The general form of Mittag-Leffler's function $\mathbb{E}_{\alpha,\beta}(z)$ is defined as [17]- [18]

$$\mathbb{E}_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\beta+\alpha k)} \tag{2}$$

where $\Gamma(.)$ is the gamma function, $\alpha$, $\beta$ are complex numbers, and $\mathrm{Re}(\alpha) > 0$. For $\beta = 1$, $\mathbb{E}_{\alpha,\beta}(z)$ reduces to ordinary Mittag-Leffler's function $\mathbb{E}_{\alpha}(z)$. Both versions are implemented in *Mathematica* as MittagLefflerE$[\alpha, \beta, z]$ and MittagLefflerE$[\alpha, z]$, respectively. Important properties of $\mathbb{E}_{\alpha,\beta}(z)$ are reported in [17]- [18].

## III. SIR Statistics

Without loss of generality, based on Slivnyak's theorem [24], we assume that the Rx of interest is located at the origin, and the Rx connects to $x_j \in \Theta_{\mathrm{T}}$ if the SIR, $\mathrm{SIR}_{x_j} = P_{\mathrm{T}_j} g_{x_j} ||x_j||^{-\eta} / \sum_{j=1}^{N_{\mathrm{I}}} \sum_{y_j \in \Theta_{\mathrm{I}_j}} P_{\mathrm{I}_j} g_{y_j} ||y_j||^{-\eta}$, is larger than a predetermined level $\gamma_0$.

Then the OP can be defined depending on a concrete operational scenario. In this work, we analyze operational protocols where the network nodes use either orthogonal channels for transmission (for example, time-division multiplexing is applied) [11], or all transmitting nodes use the same radio channel. Under the former scenarios, the outage occurs if

$$\max_{j\in[1,N_{\mathrm{T}}]} \max_{x_j \in \Theta_{\mathrm{T}_j}} \mathrm{SIR}_{x_j} \le \gamma_0, \tag{3}$$

while for the latter cases, the outage happens if

$$\sum_{j\in[1,N_{\mathrm{T}}], x_j \in \Theta_{\mathrm{T}_j}} \mathrm{SIR}_{x_j} \le \gamma_0. \tag{4}$$

### *A. MGFs of Aggregate Interference and Its Negative Fractional Power*

Proposition 1 below generalizes previously reported results on the MGF of joint interference for Nakagami-$m$ and Rayleigh fading [7]- [8].

*Proposition 1*: If raw moments $m_{l_{y_j}}$, $l = 1, \dots$ of fading distributions $\mathcal{F}_{\mathrm{I}_j}$ exist, *i.e.*, if $\lim_{b\to\infty} \int_0^b x^l f_{g_{y_j}}(x)dx$ exists for $\forall$ positive integer $l$, the MGF of joint interference $\mathcal{M}_I(-s)$ can be expressed as

$$\mathcal{M}_I(-s) = \exp\left(-Ks^{\frac{2}{\eta}}\right) \tag{5}$$

where

$$K = \pi\Gamma\left(1 - \frac{2}{\eta}\right) L_0^{\frac{2}{\eta}} \sum_{j=1}^{N_{\mathrm{I}}} \lambda_{\mathrm{I}_j} P_{\mathrm{I}_j}^{\frac{2}{\eta}} E\left\{\left(g_{\mathrm{I}_j}\right)^{\frac{2}{\eta}}\right\}. \tag{6}$$

*Proof*: See Appendix A.

Eq. (5) reduces to MGF expressions derived in [7]- [8] for Nakagami-$m$ and Rayleigh fading, respectively. Then we consider the MGF of a random variable (RV) $I^{-\frac{2}{\eta}}$.

*Proposition 2*: If the MGF of a RV $I$ is given by (5), then the MGF of $I^{-\frac{2}{\eta}}$, $\mathcal{M}_{I^{-\frac{2}{\eta}}}(-s) = E\left\{\exp\left(-sI^{-\frac{2}{\eta}}\right)\right\}$, can be expressed in terms of Mittag-Leffler's function as

$$\mathcal{M}_{I^{-\frac{2}{\eta}}}(-s) = \mathbb{E}_{\frac{2}{\eta}}\left(-\frac{s}{K}\right). \tag{7}$$

*Proof*: See Appendix B.

### *B. OP Expressions and Upper Bounds*

If the outage is represented by (3), the OP, $P_{\mathrm{out}}(\gamma_0)$, can be evaluated in a closed form.

*Lemma 1*: Let the wireless network operating in $\mathbb{R}^2$ be in the outage if (3) holds. Let the expectations $E\{g_{\mathrm{I}_j}\}$ be finite, and the MGF of aggregate interference be given by (5). Then in interference-limited scenarios with the Poisson field of transmitters $\Theta_{\mathrm{T}}$ and Poisson field of interferers $\Theta_{\mathrm{I}}$, the outage probability can be expressed as

$$P_{\mathrm{out}}(\gamma_0) = \mathbb{E}_{\frac{2}{\eta}}(-G) \tag{8}$$

where

$$G = \gamma_0^{-\frac{2}{\eta}} \frac{\sum_{j=1}^{N_{\mathrm{T}}} \lambda_{\mathrm{T}_j} P_{\mathrm{T}_j}^{\frac{2}{\eta}} E\left\{\left(g_{x_j}\right)^{\frac{2}{\eta}}\right\}}{\Gamma\left(1-\frac{2}{\eta}\right) \sum_{j=1}^{N_{\mathrm{I}}} \lambda_{\mathrm{I}_j} P_{\mathrm{I}_j}^{\frac{2}{\eta}} E\left\{\left(g_{y_j}\right)^{\frac{2}{\eta}}\right\}}. \tag{9}$$

*Proof*: See Appendix C.

The moments of fading coefficients in (6)-(9) are known for a large variety of fading models. Closed-form expressions are known, *e.g.*, for generalized $\kappa-\mu$, $\eta-\mu$ [20], and $\alpha-\mu$ [21] (also known as the Stacy or generalized gamma (GG) [22]) fading distributions. If this is not the case, these moments can easily be evaluated numerically, and for some operational scenarios, their effects are canceled.

*Corollary 1* : Fading effects in (8) are canceled if all fading distributions are identical, that is if $\mathcal{F}_{\mathrm{T}_j} = \mathcal{F}, j = 1, \dots, N_{\mathrm{T}}$, and $\mathcal{F}_{\mathrm{I}_j} = \mathcal{F}, j = 1, \dots, N_{\mathrm{I}}$.

*Corollary 2* : Effects of $L_0$ in (8) are canceled.

For the outage represented by (4), (8) provides an OP upper upper bound since for any set of positive RVs $\{a_j\}|_{j=1}^N$, $\max_{j\in[1,N]} a_j \le \sum_{j=1}^N a_j$, and thus $Pr\Big\{\max_{j\in[1,N_{\mathrm{T}}]} \max_{x_j\in\Theta_{\mathrm{T}_j}} \mathrm{SIR}_{x_j} \le \gamma_0\Big\} \ge Pr\Big\{\sum_{j\in[1,N_{\mathrm{T}}], x_j\in\Theta_{\mathrm{T}_j}} \mathrm{SIR}_{x_j} \le \gamma_0\Big\}$.

## IV. Numerical Results

In this section, we present numerical estimates obtained with the help of (8) and via Monte Carlo simulations. We considered circular operational areas and generated PPPs of the density $\lambda$ based on [25, Theorem 5] as $N$ uniformly distributed spatial points in the circle of radius $R$ where $N$ followed a Poisson distribution with the density $\pi R^2 \lambda$.

We analyzed scenarios where the useful and interference links followed Nakagami-$m$ and GG fading distributions [23] with different fading parameters. The three-parameter GG fading distribution is a general fading model including the Nakagami-$m$ and Weibull [19] fading models as particular cases and log-normal distribution [19] as a limiting case [23]. The PDF of a GG RV $g_{\mathrm{GG}}$ can be represented as

$$f_{g_{\mathrm{GG}}}(x) = \frac{\nu(\beta/\bar{g})^{m\nu}}{\Gamma(m)} x^{\nu m-1} \exp\left[-\left(\frac{\beta x}{\bar{g}}\right)^{\nu}\right] \tag{10}$$

where $m$ and $\nu$ are shape parameters, $\beta = \Gamma(m+1/\nu)/\Gamma(m)$, and $\bar{g} = E\{g_{\mathrm{GG}}\}$ [23].

We evaluated the OP for $\gamma_0 = 0$ dB versus the parameter $\lambda_{\mathrm{I}_1}/\lambda_{\mathrm{T}_1}$ for a two-tier field of interferers and for one- and two-tier fields of Txs. Network parameters were $\lambda_{\mathrm{I}_2} = 0.1\lambda_{\mathrm{T}_1}$, $\lambda_{\mathrm{T}_2} = 0.1\lambda_{\mathrm{T}_1}$, $P_{\mathrm{T}} = P\{1;2\}$, and $P_{\mathrm{I}} = P\{100;1\}$.

Numerical estimates for scenarios with identical link fading coefficients are shown in Fig. 1. For different parameters of Nakagami-$m$ and GG models, our simulations confirmed that the OP (8) was independent of fading distribution. Moreover, the simulations revealed that the OP with the outage defined by (4) was independent of fading distribution too.

Next, we analyzed the OP in a fading environment where the useful and interference links followed the GG distribution (10) with different parameters. In each analyzed set up, the shape parameter $m$ was the same for the useful and interfering links, and we tested values $m = 1.6$ and $m = 3.5$. The shape parameters $\nu$ were different for the useful and interfering links. For all considered scenarios, we assumed ordinary Nakagami-$m$ fading for the useful links (that is, $\nu_x = \{1;1\}$) while for interfering links, we used two sets $\nu_y = \{2,2\}$ and $\nu_y = \{6,4\}$. In Fig. 2, we show OP estimates for two-tier fields of Txs and interferers. Both shape parameters $m$ and $\nu$ are inversely proportional to the amount of fading [19], and thus increasing of $\nu_y$ for a fixed $\nu_x$ makes interference effects more evident. Increasing of the shape parameter $m$ boosts both the useful and interfering powers. For the used parameters, we observed some decreasing of $P_{\mathrm{out}}$ as $m$ increased.

Generally, (8) can conveniently be applied to specify the OP sensitivity to any system parameter since properties of Mittag-Leffler's function have been studied well [17]- [18].

## V. Conclusion

In this letter, we analyzed SIR statistics for wireless networks operating over arbitrary fading radio channels in interference-limited scenarios with heterogeneous Poisson fields of transmitters and interferers. We derived simple formulas for the moment generating functions of aggregate interference, its negative fractional power, and outage probability. All formulas are directly expressed via network and fading parameters.

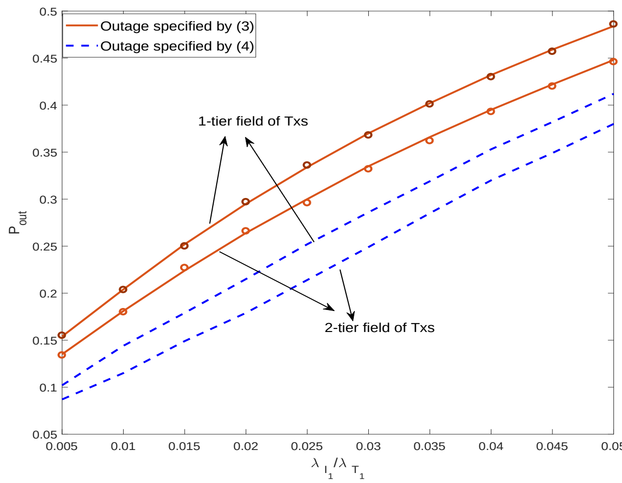


Fig. 1: Outage probability for two-tier fields of interferers and one- and two-tier fields of Txs with identical fading distributions of useful and interfering links. Single points report simulation results for the outage specified by (3).

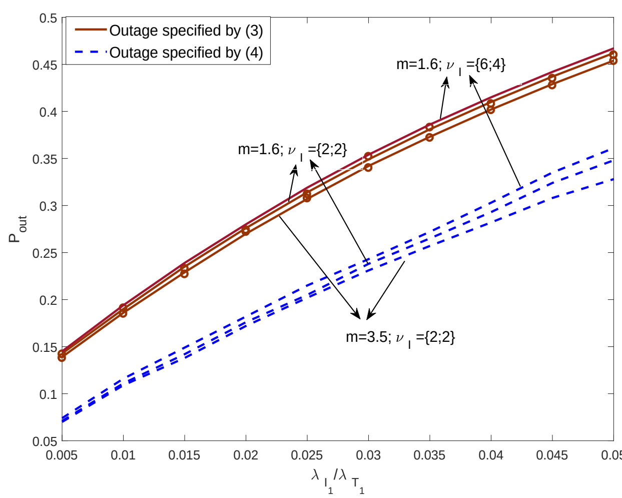


Fig. 2: Outage probability for two-tier fields of Txs and interferers with different generalized gamma fading distributions of useful and interfering links. Single points report simulation results for the outage specified by (3).

Some results of this letter were obtained in terms of Mittag-Leffler's function. This special function is implemented in modern software, which makes its application convenient.

## Appendix A
## Proof of Proposition 1

Since the PPPs $\Theta_{\mathrm{I}_j}$ are independent, and fading coefficients $g_{y_j}$ are independent too, $\mathcal{M}_I(-s)$ can be defined as

$$\mathcal{M}_I(-s) = \prod_{j=1}^{N_{\mathrm{I}}} E_{y_j \in \Theta_{\mathrm{I}_j}, g_{y_j}}$$

$$\left\{ \prod_{y_j \in \Theta_{\mathrm{I}_j}} \exp\left(-s L_0 P_{\mathrm{I}_j} g_{y_j} ||y_j||^{-\eta}\right) \right\}$$

$$\overset{(a)}{=} \prod_{j=1}^{N_{\rm I}} E_{y_j \in \Theta_{{\rm I}_j}} \left\{ \prod_{y_j \in \Phi_{{\rm I}_j}} \mathcal{M}_{g_{y_j}} \left( -sL_0 P_{{\rm I}_j} ||y_j||^{-\eta} \right) \right\} \overset{(b)}{=}$$
$$\prod_{j=1}^{N_{\rm I}} \exp\left( -\lambda_{{\rm I}_j} \int_{\mathbb{R}^2} \left[ 1 - \mathcal{M}_{g_{y_j}} \left( -sL_0 P_{{\rm I}_j} ||y_j||^{-\eta} \right) \right] dy_j \right)$$
$$\overset{(c)}{=} \prod_{j=1}^{N_{\rm I}} \exp\Bigg( - \lambda_{{\rm I}_j} \pi \left( sL_0 P_{{\rm I}_j} \right)^{\frac{2}{\eta}}$$
$$\times \underbrace{\int_0^\infty \frac{2}{\eta} t^{-\frac{2}{\eta}-1} \left[ 1 - \mathcal{M}_{g_{y_j}}(-t) \right] dt}_{{\rm Int}_{y_j}} \Bigg) \tag{11}$$

where (a) is due to the fact that the expectations with respect to $||y_j||$ and $g_{y_j}$ can be evaluated sequentially since these RVs are independent, (b) results from the probability generating functional (PGF) of PPP [2], and (c) follows from algebraic manipulations after converting from Cartesian to polar coordinates $\{r, \theta\}$ and substituting $t = sL_0 P_{{\rm I}_j} r^{-\eta}$.

The integral ${\rm Int}_{y_j}$ in (11) can be evaluated by parts. Using a series expansion $\mathcal{M}_{y_j}(-t) = \sum_{l=0}^{\infty} \frac{(-t)^l m_{l_{y_j}}}{l!}$, it can easily be shown that $\lim_{t \to 0} t^{-\frac{2}{\eta}} \left[ 1 - \mathcal{M}_{g_{y_j}}(-t) \right] = 0$ since $\frac{2}{\eta} < 1$ and the moments $m_{l_{y_j}}$ are assumed to be finite. Then taking into account that $\frac{d\mathcal{M}_y(-t)}{dt} = -E\{y \exp(-ty)\}$, we find that

$${\rm Int}_j = \int_0^\infty t^{-\frac{2}{\eta}} E\{y_j \exp(-ty_j)\} dt$$
$$\overset{(a)}{=} \int_0^\infty y f_{g_{y_j}}(y) \int_0^\infty t^{-\frac{2}{\eta}} \exp(-ty) dt dy \overset{(b)}{=} \Gamma\left(1 - \frac{2}{\eta}\right)$$
$$\times \int_0^\infty y^{\frac{2}{\eta}} f_{g_{y_j}}(y) dy = \Gamma\left(1 - \frac{2}{\eta}\right) E\left\{ g_{y_j}^{\frac{2}{\eta}} \right\} \tag{12}$$

where (a) follows from representation of $E\{y_j \exp(-ty_j)\}$ in an integral form and Fubini's theorem [26], and (b) results from an expression of the inner integral in terms of gamma function [27, vol. 1, eq. (2.3.18.2)].

Then (5) immediately follows from (11)-(12). ■

## Appendix B
## Proof of Proposition 2

Using a series expansion of exponential function, we note that

$$E\left\{ \exp\left( -sI^{-\frac{2}{\eta}} \right) \right\} = 1 + E\left\{ \sum_{m=1}^{\infty} \frac{(-s)^m}{m!} I^{-\frac{2m}{\eta}} \right\}$$
$$\overset{(a)}{=} 1 + \sum_{m=1}^{\infty} \frac{(-s)^m}{m!} E\left\{ I^{-\frac{2m}{\eta}} \right\} \overset{(b)}{=} 1 + \sum_{m=1}^{\infty} \frac{(-s)^m}{m! \Gamma\left(\frac{2m}{\eta}\right)}$$
$$\times E\left\{ \int_0^\infty \exp\left( -t \cdot I \right) t^{\frac{2m}{\eta}-1} dt \right\}$$
$$\overset{(c)}{=} 1 + \sum_{m=1}^{\infty} \frac{(-s)^m}{m! \Gamma\left(\frac{2m}{\eta}\right)} \int_0^\infty t^{\frac{2m}{\eta}-1} \underbrace{\mathcal{M}_I(-t)}_{\exp\left(-Ks^{\frac{2}{\eta}}\right)} dt$$
$$\overset{(d)}{=} 1 + \frac{\eta}{2} \sum_{m=1}^{\infty} \frac{\left(-\frac{s}{K}\right)^m \Gamma(m)}{m! \Gamma\left(\frac{2m}{\eta}\right)}$$
$$\overset{(e)}{=} 1 + \sum_{m=1}^{\infty} \frac{\left(-\frac{s}{K}\right)^m}{\Gamma\left(1 + \frac{2m}{\eta}\right)} \triangleq \mathbb{E}_{\frac{2}{\eta}}\left(-\frac{s}{K}\right) \tag{13}$$

where (a) follows from the absolute convergence of the series under the expectation operator [24]; (b) follows from a representation of $I^{-\frac{2m}{\eta}}$ in terms of gamma function as $I^{-\frac{2m}{\eta}} = \frac{1}{\Gamma\left(\frac{2m}{\eta}\right)} \int_0^\infty \exp\left(-t \cdot I\right) t^{\frac{2m}{\eta}-1} dt$ [27, vol. 1, eq. (2.3.18.2)]; (c) follows from Fubini's theorem [26]; (d) follows from the MGF expression of aggregate interference (3) and an integral representation of gamma function [27, vol. 1, eq. (2.3.18.2)]; (e) results from a property of gamma function such that $\Gamma(z+1) = (z)\Gamma(z)$ [27, vol. 3, II.3]. ■

## Appendix C
## Proof of Lemma 1

The OP, $P_{\rm out}(\gamma_0) = Pr\left\{ \max_{x \in \Theta_{\rm T}} {\rm SIR}_x \le \gamma_0 \right\}$, can be evaluated as

$$P_{\rm out}(\gamma_0) = Pr\left\{ \max_{j \in [1, N_{\rm T}]} \max_{x_j \in \Theta_{T_j}} P_{{\rm T}_j} L_0 g_{x_j} ||x_j||^{-\eta} \le \gamma_0 I \right\}$$
$$\overset{(a)}{=} E_{\rm I}\left\{ \prod_{j=1}^{N_{\rm T}} \prod_{x_j \in \Theta_{T_j}} F_{g_{x_j}} \left[ \gamma_0 (P_{\rm T} L_0)^{-1} I ||x_j||^{\eta} \right] \right\}$$
$$\overset{(b)}{=} E_{\rm I}\Bigg\{ \exp\Bigg( - \sum_{j=1}^{N_{\rm T}} \lambda_{{\rm T}_j} p_{{\rm T}_j}$$
$$\times \underbrace{\int_{x_j \in \mathbb{R}^2} \left[ 1 - F_{g_{x_j}} \left( \gamma_0 (P_{{\rm T}_j} L_0)^{-1} I ||x_j||^{\eta} \right) \right] dx_j}_{{\rm Int}_{x_j}} \Bigg) \Bigg\} \tag{14}$$

where (a) follows from independence of RVs $I$, $||x_j||$, $g_{x_j}$, and evaluation of the expectation w.r.t. $g_{x_j}$ via the CDF of $\mathcal{F}_{{\rm T}_j}$; (b) results from the PGF of PPP [2].

To evaluate the integral ${\rm Int}_{x_j}$ in (14), we convert from Cartesian to polar coordinates $(r; \theta)$ and substitute $t = \gamma_0 (P_{\rm T} L_0)^{-1} I r^{\eta}$. Then we find that

$${\rm Int}_{x_j} = \frac{2\pi}{\eta} I^{-\frac{2}{\eta}} \left( \frac{P_{{\rm T}_j} L_0}{\gamma_0} \right)^{\frac{2}{\eta}} \int_0^\infty r^{\frac{2}{\eta}-1} [1 - F_{g_{x_j}}(r)] dr$$
$$= \pi I^{-\frac{2}{\eta}} \left( \frac{P_{{\rm T}_j} L_0}{\gamma_0} \right)^{\frac{2}{\eta}} \underbrace{\int_0^\infty r^{\frac{2}{\eta}} f_{g_{x_j}}(r) dr}_{E\left\{ \left(g_{x_j}\right)^{\frac{2}{\eta}} \right\}} \tag{15}$$

where we applied integration by parts. We note that $\lim_{r \to \infty} r^{\frac{2}{\eta}} f_{x_j}(r) = 0$ since $\frac{2}{\eta} < 1$ and $\lim_{r \to \infty} r f_{x_j}(r) = 0$. The latter condition is a necessary condition of convergence of the integral $\int_0^\infty f_{x_j}(r) dr = 1$ [28].

Then plugging (15) into (14), we find that

$$P_{\rm out}(\gamma_0) = E_{\rm I}\Bigg\{ \exp\Bigg[ -I^{-\frac{2}{\eta}} \pi$$
$$\times \sum_{j=1}^{N_{\rm T}} \left( \frac{P_{{\rm T}_j} L_0}{\gamma_0} \right)^{\frac{2}{\eta}} E\left\{ \left( \gamma_{{\rm T}_j} \right)^{\frac{2}{\eta}} \right\} \Bigg] \Bigg\}. \tag{16}$$

Finally, using (5)-(6), we obtain (8). ■

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